Exercise 96

The radius r, in inches, of a spherical balloon is related to the volume, V, by $r(V) = \sqrt[3]{\frac{3V}{4\pi}}$. Air is pumped into the balloon, so the volume after t seconds is given by $V(t) = 10 + 20t$.

(a) Find the composite function $r(V(t))$.

(b) Find the *exact* time when the radius reaches 10 inches.

Solution

Plug the formula for $V(t)$ into the formula for $r(V)$.

$$\begin{aligned} r(V(t)) &= \sqrt[3]{\frac{3V(t)}{4\pi}} \\ &= \sqrt[3]{\frac{3(10+20t)}{4\pi}} \end{aligned}$$

Therefore,

$$\boxed{r(V(t)) = \sqrt[3]{\frac{30(1+2t)}{4\pi}}.}$$

To get the time when the radius reaches 10 inches, set $r = 10$ and solve the equation for t.

$$10 = \sqrt[3]{\frac{30(1+2t)}{4\pi}}$$

Cube both sides.

$$10^3 = \frac{30(1+2t)}{4\pi}$$

Multiply both sides by 4π.

$$1000(4\pi) = 30(1+2t)$$

Divide both sides by 30.

$$\frac{4000}{30}\pi = 1 + 2t$$

Subtract both sides by 1.

$$\frac{400}{3}\pi - 1 = 2t$$

Therefore, dividing both sides by 2,

$$\boxed{t = \frac{1}{2}\left(\frac{400}{3}\pi - 1\right) \approx 209 \text{ seconds.}}$$